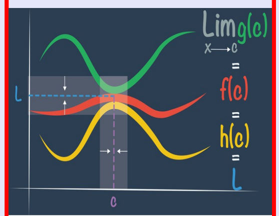
Calculus I

Lecture 40



Feb 19-8:47 AM

Class QZ 14

$f(x) = x^3 - 6x^2 - 15x + 4$

Find the x-value of all Critical Points and possible inflection Points.

$f'(x) = 3x^2 - 12x - 15$

$f'(x) = 0$

$3x^2 - 12x - 15 = 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x - 5 = 0 \qquad x + 1 = 0$

$\boxed{x = 5} \qquad \boxed{x = -1}$

$(5, f(5)) \qquad (-1, f(-1))$

$f''(x) = 6x - 12$

$f''(x) = 0$

$6x - 12 = 0$

$\boxed{x = 2}$

P.I.P. $(2, f(2))$

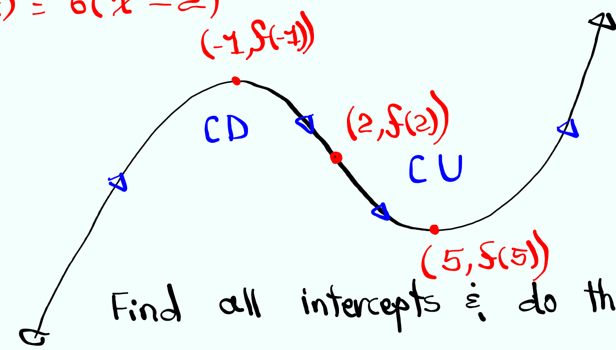
Nov 7-8:17 AM

x	$-\infty$		-1		2		5		∞
$f'(x)$		$+$		$-$		$-$		$+$	
$f''(x)$		$-$		$-$		$+$		$+$	
$f(x)$		↗		↘		↘		↗	

Max $(-1, f(-1))$ I.P. $(2, f(2))$ Min $(5, f(5))$

$f'(x) = 3(x+1)(x-5)$

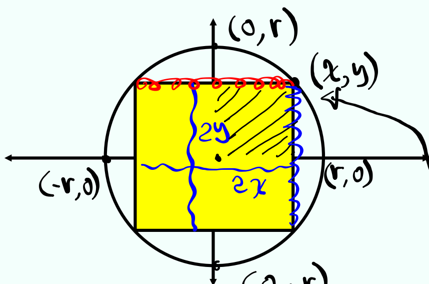
$f''(x) = 6(x-2)$



Find all intercepts & do the graph

Nov 12-7:30 AM

Find the $\boxed{\text{largest area}}$ of a rectangle inscribed in a Circle of radius r.



$L = 2x$, $W = 2y$

$A = 4xy$

$A(x,y) = 4xy$

Circle $x^2 + y^2 = r^2$

$y^2 = r^2 - x^2$

$y = \sqrt{r^2 - x^2}$

Area $\rightarrow f(x) = 4x \cdot \sqrt{r^2 - x^2}$

$f'(x)$, C.P.

$f''(x)$, $f''(\text{C.P.})$

$f''(\text{C.P.}) > 0$ Min

$f''(\text{C.P.}) < 0$ Max

Nov 7-7:50 AM

Area → $f(x) = 4x \cdot \sqrt{r^2 - x^2}$

$f(x) = 4x \cdot (r^2 - x^2)^{1/2}$

$f'(x) = 4\left[1(r^2 - x^2)^{1/2} + x \cdot \frac{1}{\cancel{2}}(r^2 - x^2)^{-1/2} \cdot (\cancel{-2}x)\right]$

$f'(x) = 4\left[(r^2 - x^2)^{1/2} - x^2(r^2 - x^2)^{-1/2}\right]$

$= 4(r^2 - x^2)^{-1/2}\left[(r^2 - x^2)^1 - x^2\right]$

$f'(x) = \dfrac{4(r^2 - 2x^2)}{\sqrt{r^2 - x^2}}$ $f'(x) = 0$ $r^2 - 2x^2 = 0$ $x = \frac{r}{\sqrt{2}}$

$f'(x)$ und. $x = r$

$r^2 - 2x^2 > 0$ $r^2 - 2x^2 < 0$

$x = 0$ $x = r$

$f'(x) > 0$ $\frac{r}{\sqrt{2}}$ $f'(x) < 0$

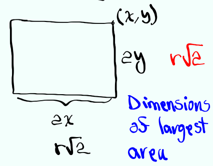
$x = \frac{r}{\sqrt{2}} = \frac{r\sqrt{2}}{2}$

$2x = r\sqrt{2}$

$x^2 + y^2 = r^2$

$\frac{r^2}{2} + y^2 = r^2$

$y^2 = \frac{r^2}{2}$ $y = \frac{r}{\sqrt{2}}$

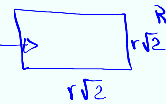


Dimensions of largest area Rectangle

$A = 4xy$

$= 4 \cdot r\sqrt{2} \cdot r\sqrt{2}$

$= 8r^2$

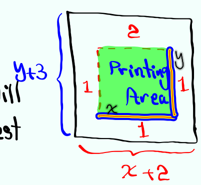


Google First & Second derivative Tests

Nov 12-7:40 AM

A poster in rectangular shape has an area of $\boxed{180 \text{ in}^2}$ with 1-inch margins at bottom and sides but 2-inch margin at the top.

What dimensions will give us the largest Printing Area?



xy

Printing Area

Maximize

$\text{Area}_{\text{Poster}} = (x+2)(y+3) = 180$

$xy + 3x + 2y + 6 = 180$

we need to maximize xy

$xy + 2y = 180 - 6 - 3x$

$y(x+2) = 174 - 3x$

$y = \dfrac{174 - 3x}{x+2}$

$f(x) = x\left(\dfrac{174 - 3x}{x+2}\right)$

$f'(x)$, C.P. $f''(x)$, $f''(\text{C.P.})$

$f''(CP) > 0$ ∪ Min

$f''(CP) < 0$ ∩ Max

Nov 7-7:58 AM

$f(x) = x\left(\frac{174-3x}{x+2}\right)$

$f(x) = -3x\cdot\left(\frac{x-58}{x+2}\right)$

$f'(x) = -3\left[1\cdot\frac{x-58}{x+2} + x\cdot\frac{1(x+2)-(x-58)\cdot 1}{(x+2)^2}\right]$

$f'(x) = -3\left[\frac{x-58}{x+2} + \frac{60x}{(x+2)^2}\right]$

$f'(x) = -3\left[\frac{(x-58)(x+2)+60x}{(x+2)^2}\right]$

$f'(x) = -3\left[\frac{x^2+2x-58x-116+60x}{(x+2)^2}\right]$ $\quad f'(x) = -3\cdot\frac{x^2+4x-116}{(x+2)^2}$

$f'(x)=0 \quad x^2+4x-116=0$ $\quad x^2+4x+4=116+4$

$f'(x)$ und. $\quad x+2=0$ $\quad (x+2)^2=120$

$x+2=\pm\sqrt{120}$

$x+2 \approx \pm 11$

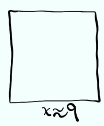
$x=-2\pm 11$

$x\approx 9 \quad \cancel{x\approx -13}$

$x=0 \qquad x=10$

$f'(x)>0 \quad 9 \quad f'(x)<0$

Max.

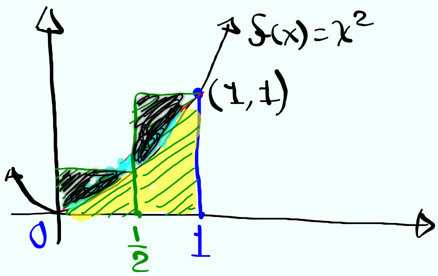


$y=\frac{174-3(9)}{9+2}$

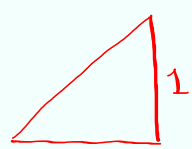
$y\approx$

Nov 12-7:55 AM

Find the area below $f(x)=x^2$, above x-axis from $x=0$ to $x=1$.

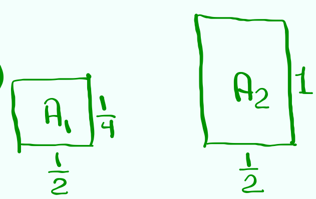


1)



$A=\frac{bh}{2}=\frac{1\cdot 1}{2}=\frac{1}{2}$

2)

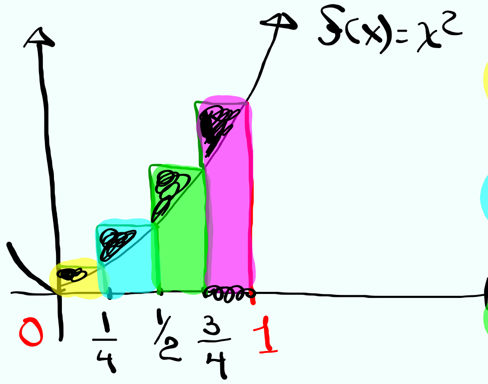


$A_1=\frac{1}{2}\cdot\frac{1}{4}$ $\qquad A_2=\frac{1}{2}\cdot 1$

$=\frac{1}{8}$ $\qquad =\frac{1}{2}=\frac{4}{8}$

$A=A_1+A_2=\frac{5}{8}$

Nov 12-8:08 AM



$A_1 = \frac{1}{4}\cdot\left(\frac{1}{4}\right)^2 = \frac{1}{64}$

$A_2 = \frac{1}{4}\left(\frac{1}{2}\right)^2 = \frac{1}{16}$

$A_3 = \frac{1}{4}\cdot\left(\frac{3}{4}\right)^2 = \frac{9}{64}$

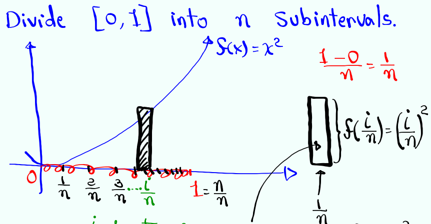
$A_4 = \frac{1}{4}(1)^2 = \frac{1}{4}$

Total Area $= A_1 + A_2 + A_3 + A_4$

$= \frac{1}{64} + \frac{1}{16} + \frac{9}{64} + \frac{1}{4} = \frac{15}{32}$

Nov 12-8:14 AM

Divide $[0,1]$ into n Subintervals.



$\frac{1-0}{n} = \frac{1}{n}$

$i=1$ to n

Let $n \to \infty$

$A_i = \frac{1}{n}\cdot\left(\frac{i}{n}\right)^2 = \frac{i^2}{n^3}$

Total Area $= A_1 + A_2 + A_3 + \cdots + A_i + \cdots A_n$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} A_i = \lim_{n\to\infty} \sum_{i=1}^{n} \frac{i^2}{n^3}$$

$$= \lim_{n\to\infty} \frac{1}{n^3} \sum_{i=1}^{n} i^2$$

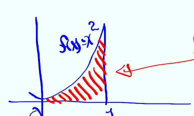
$$= \lim_{n\to\infty} \frac{1}{n^3}\cdot\frac{n(n+1)(2n+1)}{6}$$

$$= \lim_{n\to\infty} \frac{2n^3 + \cancel{\text{Junk}}^{\to 0}}{6n^3}$$

$$= \frac{2}{6} = \boxed{\frac{1}{3}}$$

$1+2+3+4+\cdots+n = \frac{n(n+1)}{2}$

$1^2+2^2+3^2+4^2+\cdots+n^2 = \frac{n(n+1)(2n+1)}{6}$



Area $= \frac{1}{3}$

Nov 12-8:22 AM